[Total No. of Questions: 09]
Uni. Roll No. ...............

[Total No. of Pages: 02]

Program: B.Tech. (Batch 2018 onward)
Semester: 4th
Name of Subject: Mathematics - III
Subject Code: BSME-101
Paper ID: 16197
Scientific calculator is Allowed

**MORNING**
**19 SEP 2022**

Time Allowed: 03 Hours

Max. Marks: 60

**NOTE:**

1) Parts A and B are compulsory
2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
3) Any missing data may be assumed appropriately

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**Part – A**      **[Marks: 02 each]**

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**Q1.**

a) Classify the partial differential equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$.

b) Discuss the nature of singularity of $f(z) = \sin \dfrac{1}{z}$ at $z = 0$.

c) Find the coefficient of correlation when regression coefficient of $X$ on $Y$ is -0.37 and regression coefficient of $Y$ on $X$ is -0.73.

d) Suppose an ideal die is tossed twice. What is the probability of getting a sum 10 in two tosses.

e) Evaluate $\oint_C \dfrac{z^2 + 5}{z - 3} \, dz$, where $C$ is the circle $|z| = 4$.

f) Find the particular integral of $(D^2 + 5DD' + 6D'^2)z = \dfrac{1}{y - 2x}$.

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**Part – B**      **[Marks: 04 each]**

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**Q2.** Solve $z(x + y)p + z(x - y)q = x^2 + y^2$.

**Q3.** If $u = e^x(x \cos y - y \sin y)$ is a harmonic function, find an analytic function $f(z) = u + iv$.

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**Q4.** Solve $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial^2 z}{\partial x \partial y} + \dfrac{\partial^2 z}{\partial y^2} = \sin x$.

**Q5.** Obtain the Laurent's series for $\dfrac{1}{z(z^2 - 3z + 2)}$ in $1 < |z| < 2$.

**Q6.** Sample of sizes 10 and 14 were taken from two normal populations with standard deviations 3.5 and 5.2. The sample mean were found to be 20.3 and 18.6. Test whether the means of two populations are the same at 5% level of significance.

**Q7.** A car hire firm has two cars which it hires out day to day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days,

   a) on which there is no demand

   b) on which demand is refused ($e^{-1.5} = 0.2231$).

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**Part – C**                       **[Marks: 12 each]**

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**Q8.** Use method of separation of variables to find the solution of heat conduction equation

$\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$, $0 \le x \le 1$, $t \ge 0$ with boundary conditions $u(0,t) = u(1,t) = 0$ and initial

conditions $u(x,0) = x(1-x)$.        **OR**

If $X$ is a normal variable with mean 25 and standard deviation 5, then Find

(i) $P(X \le 10)$       (ii) $P(15 \le X \le 30)$       (iii) $P(|X - 30|) \ge 10$.

**Q9.** Evaluate the integral $\displaystyle\int_0^{2\pi} \dfrac{d\theta}{5 - 3\cos\theta}$.

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**OR**

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A set of five similar coins is tossed 320 times and the result is

| No. of heads | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequencies | 6 | 27 | 72 | 112 | 71 | 32 |

Test the hypothesis that the data follows binomial distribution.

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